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We present results from a study of meson spectra and structure in the limit where one quark is infinitely heavy. The calculations, based on the framework of light-front QCD formulated on a transverse lattice, are the first non-perturbative studies of B-mesons in light-front QCD. We calculate the Isgur-Wise form factor, light-cone distribution amplitude, the heavy-quark parton distribution function and the leptonic decay constant of B-mesons.

I. INTRODUCTION

Heavy flavor physics has become one of our most promising tools in searching for physics beyond the standard model. However, despite the use of heavy quark symmetry and heavy quark effective field theory, many uncertainties in extracting standard model parameters from data on heavy meson decays arise from the often unknown nonperturbative hadronic matrix elements for these decays. For example, in order to extract the CKM matrix matrix element $|V_{bc}|$ from semileptonic $\bar{B} \rightarrow D^*$ transitions, it is necessary to extrapolate to the point $v \cdot v' = 1$. Even in the heavy quark limit, this involves the Isgur-Wise form factor, which is not determined by symmetry. For this and similar applications, it is essential to have reliable predictions for hadronic matrix elements that multiply interesting standard model parameters in the physical amplitudes.[1]

Euclidean lattice gauge theories have played an important role for calculating some of the relevant nonperturbative matrix elements. However, especially for matrix elements that involve a large velocity transfer as well as matrix elements that probe light-like correlation functions (such as the B-meson distribution amplitude or the parton distribution functions, which are relevant for inclusive semileptonic decays), it would be much more natural to describe the nonperturbative hadronic physics in the light-front (LF) framework (or infinite momentum frame), i.e. in a Hamiltonian framework where $x^+ \equiv x^0 + x^3$ is 'time'.

What distinguishes the LF framework from all other formulation of QCD is that the observables, which are dominated by light-like correlation functions, become equal 'time' correlation functions and hence have a very direct and physical connection to the microscopic degrees of freedom in terms of which the Hamiltonian is constructed [3]. Because of this unique feature, it should be much easier in this framework to gain a physical understanding between experiment and phenomenology on the one hand and the underlying QCD dynamics on the other.

Of course, just like in any other approach to QCD, it is necessary to regularize both UV and IR divergences before one can even attempt to perform nonperturbative calculations. The transverse lattice [4] is an attempt to combine advantages of the LF and lattice formulations of QCD. In this approach to LF-QCD the time and one space direction (say x^3) are kept continuous, while the two 'transverse' directions $\mathbf{x}_\perp \equiv (x^1, x^2)$ are discretized. Keeping the time and x^3 directions continuous has the advantage of preserving manifest boost invariance for boosts in the x^3 direction. Furthermore, since $x^\pm = x^0 \pm x^3$ also remain continuous, this formulation still allows a canonical LF Hamiltonian approach. On the other hand, working on a position space lattice in the transverse direction allows one to introduce a gauge invariant cutoff on $\perp$ momenta — in a manner that is similar to Euclidean or Hamiltonian lattice gauge theory.

In summary, the LF formulation has the advantage of utilizing degrees of freedom that are very physical since many high-energy scattering observables (such as deep-inelastic scattering cross sections) have very simple and intuitive interpretations as equal LF-time (x^+) correlation functions. Using a gauge invariant (position space-) lattice cutoff in the $\perp$ direction within the LF framework has the advantage of being able to avoid the notorious $1/k^+$ divergences from the gauge field in LF-gauge which plague many other Hamiltonian LF approaches to QCD [5].

The hybrid (continuous along with discrete) treatment of the longitudinal/transverse directions implies an analogous hybrid treatment of the longitudinal versus transverse gauge field: the longitudinal gauge field degrees of freedom are the non-compact A^μ while the transverse gauge degrees of freedom are compact link-fields. Each of these degrees of freedom depend on two continuous ($x^\pm$) and two discrete ($\mathbf{n}_\perp$) space-time variables, i.e. from a formal point of view the canonical transverse lattice formulation is equivalent to a large number of coupled $1+1$ dimensional gauge theories (the longitudinal gauge fields at each $\mathbf{n}_\perp$) coupled to nonlinear σ model degrees of freedom (the link fields) [6].

[1]For a recent review, see Ref. [1].

This formalism has been previously applied to pure-glue QCD (numerical results for the static quark-antiquark potential can for example be found in Ref. [7] and for glueball spectra in Ref. [8]). More recent applications of the color dielectric transverse lattice formulation to mesons at large N_C can be found in Refs. [9–11], where more details regarding the transverse lattice approach approach can be found.

In this paper, we make the first attempt to apply the $\perp$ lattice, used in the above light meson calculations, to mesons where one of the quarks is infinitely heavy. The paper is organized as follows. We briefly describe the Hamiltonian, the approximations used to solve it in section 2. Numerical results for a number of important hadronic matrix elements are presented in section 3.

II. THE HAMILTONIAN

The transverse lattice Hamiltonian for Wilson fermions that forms the basis of our work was first constructed in Ref. [12] and was previously applied to mesons containing only light quarks in Refs. [9,11], and we refer the reader to these works for more details. Here we restrict ourselves to a more qualitative description of the degrees of freedom that enter the Hamiltonian and some of the details of how heavy quarks are incorporated in this framework.

The degrees of freedom that enter the Hamiltonian are quark and antiquark creation operators at each site and link-fields (which, in the spirit of the color-dielectric approach [8], we take as general matrix fields) on the transverse links connecting the sites. For simplicity, we work in the large N_C limit. The first approximation that we use is the same light front Tamm-Dancoff truncation of the Fock state expansion that was used in Refs. [9,11], namely, a truncation upto and including the 3-particle sector. After reminding oneself that the fermion degrees of freedom (quarks and antiquarks) always occupy the lattice (transverse) sites and the gauge degrees of freedom (link fields) reside on the lattice spacings, it is easy to understand the following two direct consequences of the aforementioned truncation:

- The 2-particle states consist of a quark and an antiquark sitting on the same lattice site.
- The 3-particle states consist of a quark and an antiquark separated by **at most** one link field.

The Hamiltonian for the light quarks and link fields in this model contains the following 6 parameters

- kinetic masses for the light quarks in the 2 and 3 particle Fock sector
- helicity flip and non-flip hopping terms for the light quarks
- longitudinal gauge coupling
- kinetic mass for the link field

which were determined in Ref. [11] by imposing rotational invariance, fixing the physical string tension in $\perp$ lattice units in both the longitudinal and transverse direction and the π and ρ masses.

For the heavy quarks one has a variety of options. Conceptually, it would be cleanest to treat them as infinitely heavy fixed sources, using the fixed source formalism outlined in Ref. [7]. In such an approach, no new parameters would enter the Hamiltonian. In this work, we adopted a slightly different procedure to implement the heavy quarks: we treated them as finite mass quarks, but did not allow the heavy quarks to propagate in the transverse direction — only propagation in the longitudinal direction is allowed. The advantage if this hybrid procedure is that we could utilize computer code that was used to describe light-light mesons and the heavy quark limit results were then obtained by extrapolation. [2] Treating heavy quarks consistently as finite quark mass heavy quarks would have the advantage that one could study $1/M_b$ corrections to the heavy quark limit. However, since this would not only require developing an analog of the NRQCD procedure on the LF, but also a determination of a whole new set of parameters (the coefficients of the hopping terms for the heavy quark in the Hamiltonian), we preferred to focus only on the $M_b \to \infty$ limit, where no new parameters are introduced.

III. NUMERICAL RESULTS

The observables that we studied are the Isgur Wise form factor, the decay constant f_B, the B-meson light cone distribution amplitude and the (light-quark) parton distribution function in a B-meson. For all these observables, we performed calculations at finite kinetic mass M_b (but no $\perp$ hopping) and then extrapolated to $M_b \to \infty$. Any error bars/bands that we show for these observables reflect uncertainties arising from this extrapolation.

In each of these calculations, we diagonalized the $\perp$ lattice Hamiltonian in a truncated basis, using continuous basis functions. As output of these calculations we obtained the wave functions (Fock space amplitudes) in the two and three particle Fock components $\psi_s(x)$ and $\psi_s(x,y)$ where the arguments in the two particle Fock component refer to the momentum fraction carried by the light quark and in the three particle Fock component refers to the momentum fractions carried by the light and

[2] Note that although this procedure produces the correct infinite quark mass results through extrapolation, we cannot use the results to study finite M_b corrections, since the dynamics of finite M_b is not properly described (no transverse hopping!).

heavy quark. The discrete index s labels the spin configurations (4 in the two particle Fock component) and the spin/orientation components ($4 \times 4 = 16$ in the three particle Fock component).

In the following, we will use numerical results for these wave functions to calculate some important hadronic matrix elements for these heavy-light systems.

One of the most important hadronic observables in B-physics is the Isgur-Wise form factor. Heavy quark symmetry predicts that the matrix element of a heavy quark current operator in heavy-light mesons is described by a universal form factor

$$\langle B(p')|\bar{b}\gamma^\mu b|B(p)\rangle = M_B \left(v^\mu + v'^\mu\right) F(v \cdot v') \qquad (3.1)$$

Although this form factor does still depend on the quantum numbers of the light degrees of freedom, it becomes independent of the heavy quark mass as $M_Q \to \infty$. Knowledge of this form factor is very useful for determining the CKM matrix element V_{bc} from semi-leptonic decays $\bar{B} \to D^* e\nu_e$ because it allows one to extrapolate decay data to the zero recoil point $v \cdot v' \to 1$. Within the context of the transverse lattice, the form factor is most conveniently extracted from the + component of the current operator, for which we find

$$F(v \cdot v') = F^{(2)}(v \cdot v') + F^{(3)}(v \cdot v'), \qquad (3.2)$$

where

$$F^{(2)}(x) = \frac{2}{2-x} \sum_s \int_x^1 dz \psi_s(z) \psi_s^* \left(\frac{z-x}{1-x}\right) \qquad (3.3)$$

$$F^{(3)}(x) = \frac{2}{2-x} \frac{1}{\sqrt{1-x}} \sum_s \int_x^1 dz \qquad (3.4)$$

$$\int_0^{1-z} dw \psi_s(z,w) \psi_s^* \left(\frac{z-x}{1-x}, \frac{w}{1-x}\right) \qquad (3.5)$$

are the contributions from the two and three particle Fock components respectively and $\sum_s$ refers to the summation over the discrete spin and orientation label for the meson. The velocity transfer is related to light-cone plus momentum transfer x through the relation $v \cdot v' = \frac{1}{2}\left(1 - x + \frac{1}{1-x}\right)$. Note that the normalization condition $F(v \cdot v' = 1) = 1$ is automatic in this framework because of the normalization condition for the wave function. Note also that terms that are off-diagonal in Fock space, and which in general have to be included when calculating current matrix elements when $q^+ \equiv p^{+'} - p^+$ is nonzero [13], are suppressed in the limit $M_b \to \infty$ (at least when considering the 'good' component).

In the numerical calculations we evaluated Eq. (3.2) for a variety of heavy quark masses and extrapolated to $M_b \to \infty$. Results for the extrapolated form factor are depicted in Fig. 1, where we multiplied the form factor with $V_{bc} = 0.0424$ [14] in order to be able to compare to experimental data [14]. Overall, our calculated form factor is in fairly good agreement with the data, although the experimental error bars are quite large. We should perhaps point out that if we include the data at larger values of $v \cdot v'$ into account then we would obtain a slightly better fit to the data for values of V_{bc} in the range $V_{bc} = 0.0415 - 0.0420$. The functional form of the fit to our calculated form factor as presented in Fig. 1 is:

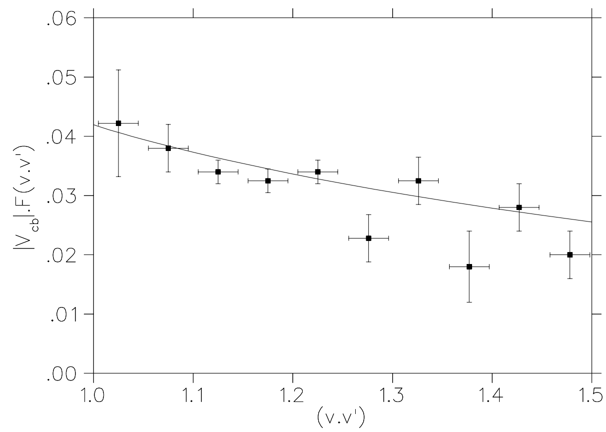

FIG. 1. Isgur-Wise form factor (3.2) in the heavy quark limit. Uncertainties from the $M_b \to \infty$ limit were very small for this observable. The experimental data is from Ref. [14].

$$F(v.v') = 1 - 1.398(v.v' - 1) + 1.722(v.v' - 1)^2$$
$$-0.203(v.v' - 1)^3 - 1.824(v.v' - 1)^4 \qquad (3.6)$$

Although no experimental data exists on

$$\langle 0|\bar{b}\gamma^\mu\gamma_5 u|B\rangle = p^\mu f_B, \qquad (3.7)$$

it would be very useful to have predictions for this observable because of its role in B-meson phenomenology.

Up to an overall scale factor which involves the $\perp$ lattice spacing, f_B can be conveniently expressed in terms of the two particle wave function on the transverse lattice [15]

$$f_B = \frac{2}{a_\perp} \sqrt{\frac{N_C}{\pi}} \int_0^1 dx \psi_{+-}(x). \qquad (3.8)$$

We find (in the heavy quark limit) for a value of $M_b = 5GeV$ and $a_\perp \approx 0.5fm$ [11]

$$f_B = 238 \pm 20\, MeV \qquad (3.9)$$

where the error bar reflects only the uncertainty in our numerical extrapolation to the heavy quark limit. A much larger systematic uncertainty in Eq. (3.9) arises due to our Fock space truncation. We expect that including more higher Fock components will significantly reduce the 2-particle Fock space amplitude and hence also the decay constant, but it is very difficult to estimate the size of the reduction.

In addition to the decay constant, we also studied the light-cone distribution amplitude itself. In the LF approach, the twist-2 distribution amplitude is given by the wave function in the two particle Fock space sector with the quark and the antiquark at the same transverse position. On the transverse lattice, the distribution amplitude, which plays a role in large momentum transfer processes, is thus conveniently expressed in terms of the two particle Fock component wavefunction $\psi(x)$

$$\phi(x) = N\psi(x), \tag{3.10}$$

where N is a normalization constant. The scaling behavior of $\phi(x)$ is such that

$$\phi_\infty(x) \equiv \lim_{M_b\to\infty} \frac{1}{\sqrt{M_b}}\phi(x/M_b) \tag{3.11}$$

scales in the heavy quark limit. Numerical results for $\phi_\infty(x)$ are shown in Fig. 2. Although a precise determination of the power law behavior near $x \to 0$ was

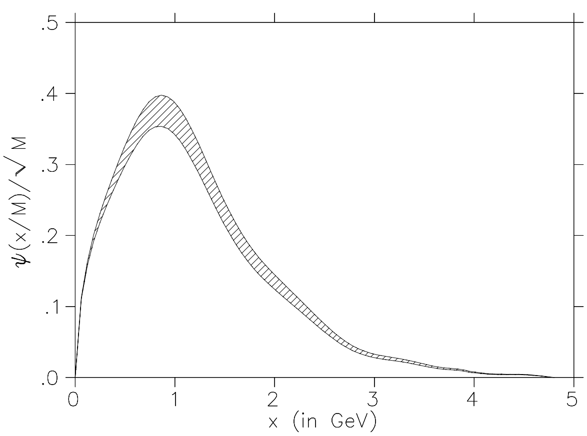

FIG. 2. B-meson distribution amplitude in the heavy quark limit. Systematic uncertainties arising from the numerical extrapolation are indicated by the shaded area.

not possible, we found that our numerical results are consistent with a power close to 0.5 below x=1 GeV. For the lowest moments, we find

$$M_{-1} \equiv \frac{\int_0^\infty \frac{dz}{z}\phi_\infty(z)}{\int_0^\infty dz\phi_\infty(z)} = 1.509\,GeV^{-1} \tag{3.12}$$

$$M_1 \equiv \frac{\int_0^\infty dz z\phi_\infty(z)}{\int_0^\infty dz\phi_\infty(z)} = 1.223\,GeV. \tag{3.13}$$

From Eqs. (3.12) and (3.13), it is clear that $\bar{\Lambda} = \frac{3}{4}M_1 \approx 0.9\,GeV$ [16] in our calculations turns out to be very large. This result is consistent with the 'binding energies' $M_B - M_b$ that we obtain and reflect the fact that the bare (unscreened) longitudinal string tension is very large. We expect that including higher Fock states will lead to a significant lowering of the longitudinal string tension and hence also $\bar{\Lambda}$.

The light-cone momentum distribution of the heavy quark inside a B-meson is relevant for the invariant mass distribution in inclusive decays of heavy-light systems because it determines the amount of energy that is available for the weak decay products of the heavy quark. Of course, to leading order in $1/M_b$ the b quark carries 100% of the B-meson's momentum in the infinite momentum frame. We thus consider the first correction to this trivial result, i.e.

$$f_b(z) = \lim_{M_b\to\infty} b\left(1 - \frac{z}{M_b}\right) \tag{3.14}$$

where $b(x)$ is the light-cone momentum distribution of the b-quark. Results are shown in Fig. 3. Since the 2 particle Fock component is the dominant component for B-mesons in our calculations, $f_b(z)$ is to a good approximation described by $|\psi_\infty(z)|^2$ (Fig. 2).

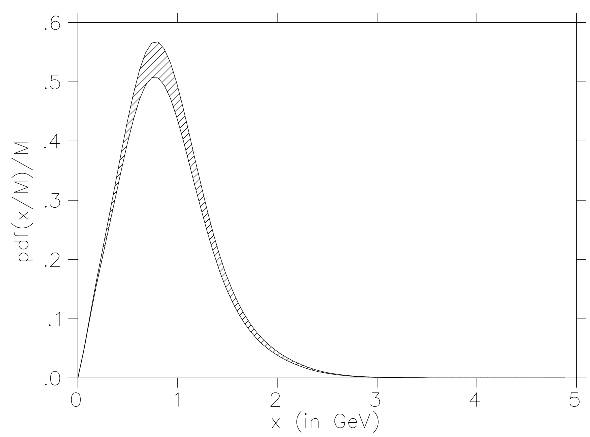

FIG. 3. Light-cone momentum distribution (3.14). The shaded area reflects uncertainties arising from the numerical extrapolation.

IV. SUMMARY AND OUTLOOK

We have performed the first non-perturbative study of mesons containing one heavy and one light quark in light front QCD based on the transverse lattice fomulation. The parameters that appear in the Hamiltonian for a heavy-light system are the same that appear also in the Hamiltonian for light-light mesons. Since we determined the latter in a previous work [11] using constraints from rotational invariance, we were thus able to make parameter free predictions for a variety of heavy meson observables.

Specifically, we studied the Isgur-Wise form factor, decay constant, light-cone distribution amplitude and light-cone momentum distribution function for B mesons in the limit $M_b \to \infty$. Our results for the form factor are consistent with experimental data and favor a CKM matrix element V_{bc} in the range $0.0415 - 0.0420$.

Our main sources of systematic errors are the Fock space truncation (in combination with a truncation of the transverse lattice Hamiltonian [9]), the truncation of the Hilbert space within each Fock component and the extrapolation to the $M_b \to \infty$ limit. The uncertainties from Hilbert space truncations are well under control since we used basis function techniques and extrapolated the results in the dimension of the Hilbert space. The $M_b \to \infty$ are also well under control since we studied a number of different M_b values. The main source of systematic errors in our calculations arise from the Fock space truncation and it is difficult to estimate the size of the corrections from higher Fock components without actually including them. Although we expect little corrections from higher Fock components for the form factor at small velocity transfers (the main effect of higher Fock component would be to provide the quarks with an additional intrinsic form factor which would only be probed at large momentum transfers), higher Fock components probably have a large effect on the decay constant which is proportional to the amplitude to find the meson in the two particle Fock component. We did not include higher Fock components in this work because, for consistency, this requires inclusion of a number of new operators in the $\perp$ lattice Hamiltonian — a project that would have been beyond the intended scope of this one. Nevertheless, it is clear that one possible future extension of this work should be to take higher Fock components into account.

There are many other natural extensions of this work. For example, it would be very useful to repeat this study for B_s mesons. While the $u\bar{b}$ meson calculations that we performed here could make use of light-quark parameters determined in previous works [11,9], such a study involving s quarks would first require determination of hopping parameters for s quarks using techniques similar to the used in Refs. [11,9].

Although we performed the numerical calculations for a variety of heavy quark masses, we did not allow the heavy quarks to propagate in the transverse directions because this would have forced us to introduce (and determine) transverse hopping parameters. The advantage of this approach was that we were able to study the $M_b \to \infty$ limit without introducing any new parameters. The disadvantage is that we cannot use the results to study the $1/M_b$ to the heavy quark observables, which would be extremely useful for the analysis of experimental data, especially those involving charm quarks.

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